Exercise 1.1 (Photon properties)

Photons are quantum particles and therefore obey the laws of quantum mechanics $E = \hbar\omega$ and $p = \hbar\boldsymbol{k}$ ($\boldsymbol{k}$ is the wavevector). For a photon with a wavelength of 500 nm, calculate its energy (J, eV), frequency ν, wavenumber $\bar{\nu}$ (cm^{-1}), momentum p and dispersion relation $\omega(\boldsymbol{k})$ in vacuum.

Exercise 1.1 Solution.

Relation between wavelength λ, frequency ν, period T, speed of light c:

$$\lambda = cT = \frac{c}{\nu}$$

We can immediately calculate frequency ν:

$$\nu = \frac{c}{\lambda} = \frac{3 \cdot 10^8 \, \frac{m}{s}}{500 \cdot 10^{-9} \, m} = 6 \cdot 10^{14} \, Hz = 600 \, THz$$

Relation between wavelength λ and wavenumber $\bar{\nu}$:

$$\bar{\nu} = \frac{1}{\lambda} = \frac{1}{500 \cdot 10^{-9} \, m} = \frac{10^7}{500} \, cm^{-1} = 20000 \, cm^{-1}$$

Energy:

$$\omega = 2\pi\nu, [\omega] = \frac{rad}{s}$$

$$E = \hbar\omega = h\nu = 6.63 \cdot 10^{-34} \, J \cdot s \cdot 6 \cdot 10^{14} \, Hz \approx 4 \cdot 10^{-19} \, J$$

In physics, an electronvolt (symbol eV, also written electron-volt and electron volt) is the measure of an amount of kinetic energy gained by a single electron accelerating from rest through an electric potential difference of one volt in vacuum. Wiki

Therefore:

$$E[eV] = \frac{E[J]}{e}$$

Where $e = 1.6 \cdot 10^{-19} \, C$ is the elementary charge.

$$E[eV] = 2.5 \, eV$$

Also, there is a convenient relation to calculate energy in eV from wavelength in nm:

$$E[eV] = \frac{1240}{\lambda[nm]}$$

$$E[eV] = \frac{1240}{500} = 2.48 \, eV$$

Relation between wavelength λ and wave vector k:

$$k = \frac{2\pi}{\lambda}$$

Therefore

$$p = \hbar k = \frac{h}{\lambda} = \frac{6.63 \cdot 10^{-34} \, J \cdot s}{500 \cdot 10^{-9} \, m} \approx 1.33 \cdot 10^{-27} \, \frac{J \cdot s}{m} = 1.33 \cdot 10^{-27} \, kg \frac{m}{s}$$

For the dispersion relation $\omega(k)$:

$$\omega = 2\pi\nu = 2\pi\frac{c}{\lambda} = 2\pi\frac{c}{\frac{2\pi}{k}} = ck$$

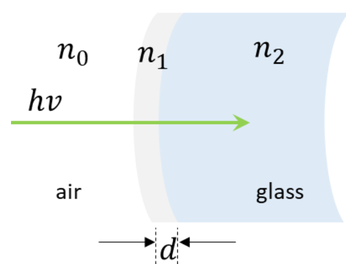
Exercise 1.2 (antireflective coating, interference)

An antireflective coating for a glass lens is designed in order to suppress reflections at a wavelength of 530 nm for perpendicular incidence. You can assume that there is no absorption in the glass and the coating.

a) Derive the minimum thickness *d* of this layer assuming a refractive index $n_1 = 1.38$ of the coating material and $n_2 = 1.5$ for glass.

b) Neglecting multiple reflections, estimate the percentage of the suppressed intensity of the reflected light due to the thin coating.
<u>Optional question</u>: Compare your result to the exact solution including multiple reflections.

c) What happens with the light that is back reflected at the uncoated interface of the glass? Can this back reflection also be suppressed? Please provide a qualitative answer.

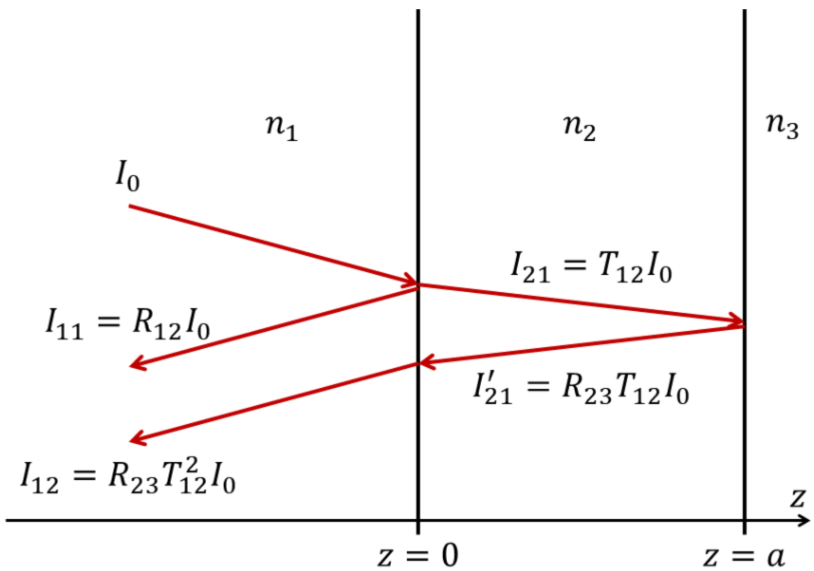


<u>Note</u>: Use $R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$ to calculate the reflectance at the n_1/n_2 interface and in analogy at the n_0/n_1 interface.

Ex. 1.2 Solution

Although the reflection coefficient from glass is relatively low (about 4% at air-glass interface with n = 1.5), in complex optical systems with many surfaces even with a small reflectance R, significant light losses eventually accumulate. Therefore, the problem of reducing the intensity of reflection from surfaces arises.

This is achieved using a thin dielectric layer deposited on the surface of the reflective medium (antireflective coating). This is shown schematically in the figure below. A flat dielectric layer (0 ≤ z ≤ a) with a refractive index $n2$ is adjacent to a reflective half-space (z > a) with a refractive index $n3$. We put a refractive index $n1$ for the left half-space ($z < 0$). However, for this particular case $n1 = 1$. This situation is shown at the following scheme. Just for clarity the picture shows the incident angle different from zero, but the calculations are made for normal incidence.



Incident wave with intensity $I0$ comes from the left half-space ($z < 0$). It reflects from the interface 1-2. The reflected wave has an intensity $I11$. We remember the link between the intensity and the amplitude: $I11 = |E11|^2$

For the coefficient of reflection for the intensity, we know:

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

So, since the incident wave has the intensity $I0$, the part of that reflects at the 1-2 interface is:

$I_{11} = R_{12}I_0$

The second part of this wave transmits to the dielectric coating ($0 \le z \le a$) with the coefficient of transmission T_{12}:

$T_{12} = 1 - R_{12} = 0.9745$

So, the transmitted wave intensity is

$I_{21} = T_{12}I_0$

The transmitted wave reaches the second interface and reflects with coefficient $R23$:

$$R_{23} = \left(\frac{n_2 - n_3}{n_2 + n_3}\right)^2 = 0.00174$$

The intensity of this wave:

$I_{21}' = R_{23} I_{21} = R_{23} T_{12} I_0$

This wave reflects at 2-1 interface with $R_{21} = R_{12} = 0.0255$ and partially transmits with $T12$ = $T21$ = 0.9745. So, the second reflected wave in medium 1 is $I12$:

$I_{12} = T_{12} I_{21}' = R_{23} T^2{}_{12}\, I_0$

We have 2 waves reflected from the lens. At this point we can use the formula for interference:

$$I = \left|E_{11} e^{i\varphi_{11}} + E_{12} e^{i\varphi_{12}}\right|^2$$

Remember that the modulus square is calculated as the product of the complex number with its conjugated version, so

$$I = I_{11} + I_{22} + 2\sqrt{I_{11} I_{22}} \cos(\Delta\varphi)$$

We assume that the coating is designed in such a way to reflect as little intensity as possible, so we seek min(I), thus $\Delta\varphi = \pi + 2\pi m$, where m = 0, ± 1, ± 2, ± 3, ….

As we seek min d => m=0, next for the phase difference of optical waves, one of which that travels 2 times through a medium:

$$\Delta\varphi = 2kd = 2 * \frac{2\pi n d}{\lambda} = \pi,$$

$$d = \frac{\lambda}{4n}$$

a) Answer: d ~ 96 nm

To estimate the percentage of the suppressed intensity of the reflected light due to the thin coating:

$$I = I_{11} + I_{22} - 2\sqrt{I_{11} I_{22}}$$

$$R = \frac{I}{I_0} = 0.0142$$

$$R_{12} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = 0.04$$

b) Answer: So, reflection suppression:

$$\frac{\Delta R}{R_{12}} = (R_{12} - R)/R_{12} = 0.65$$

c) Answer: For the uncoated back interface of the glass (glass-air interface) the reflection occurs as well. The energy reflection is the same as for air-glass interface with R = 4%. This reflection can be suppressed as well with another antireflection coating.